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WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 4th Semester Examination, 2023

PHSACOR08T-PHYSICS (CC8)

Time Allotted: 2 Hours | Full Marks: 40

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

Question No. 1 is compulsory and answer any *two* from the rest

1. Answer any ***ten*** questions from the following: $2\times 10=20$

(a) Solve $z^2(1-z^2)=16$, where z is a complex number.

(b) Find the cube roots of $(-1+i)$.

(c) Expand $f(z)=\ln(1+z)$ in a Taylor Series about $z=0$.

(d) Find the three dimensional Fourier transform of three dimensional Dirac-delta function.

(e) For a cylindrically symmetric potential ϕ, find the solution of one dimensional Laplace's equation.

(f) Show that the product of two symmetric matrices is symmetric if they commute.

(g) Evaluate e^A where matrix A is given by $A=\begin{bmatrix}1 & 0\\ 0 & 2\end{bmatrix}$.

(h) For a 2×2 square matrix A, find its eigenvalues in terms of t and d, given $\mathrm{Tr}(A)=t$ and $\det(A)=d$.

(i) If $f(s)$ is Fourier transform of $F(t)$, then show that Fourier transform of

$$F(at) \text{ is } \frac{1}{a}f\left(\frac{s}{a}\right).$$

(j) If ϕ be a function of r only, then show $\nabla^2\phi=\frac{d^2\phi}{dr^2}+\frac{2}{r}\frac{d\phi}{dr}$.

(k) Show that eigenvalues of an anti Hermitian matrix is either zero or purely imaginary.

(l) Find the Fourier sine transform of e^{-x}.

(m) Find the Fourier transform of a Dirac Delta Function $f(x)=\delta(x-a)$, 'a' being some constant.

(n) Prove that a real matrix is unitary if it is orthogonal.

2. (a) Expand $f(z)=\dfrac{1}{(z+1)(z+3)}$ in a Laurent series valid for $1<|z|<3$. 3

(b) Show that the Fourier transform of a Gaussian function is also a Gaussian function. 3

(c) An uncharged conducting sphere of radius R is placed in a uniform electrostatic field $\vec{E}=E_0\hat{k}$. Find the potential outside the sphere using solution of Laplace's equation in spherical polar coordinates. 4

3. (a) Find the characteristic equation of the matrix. 1+2+2

$$A=\begin{bmatrix}1 & 2 & 3\\ 2 & -1 & 4\\ 3 & 1 & 1\end{bmatrix}$$

and verify Cayley-Hamilton theorem for it. Hence find A^{-1}.

(b) Find the Fourier transform of the function 2

$$f(x)=\begin{cases}1 &, \text{ for } |x|<a\\ 0 &, \text{ for } |x|>a\end{cases}$$

(c) Show that $\oint_C \dfrac{e^{zt}}{z^2+1}dz = 2\pi i \sin t$, if $t>0$ and C is the circle $|z|=3$. 3

4. (a) Two matrices A and B satisfy $(AB)^T + B^{-1}A = 0$. Prove that if B is orthogonal, then A is anti-Symmetric. 3

(b) If a matrix B commutes with a diagonal matrix A, no. two elements of which are equal, show that, B is a diagonal matrix. 2

(c) For the following function locate and name the singularities in the finite z-plane and determine whether they are isolated singularities or not. 5

$$f(z)=\frac{z}{(z^2+4)^2}$$

5. (a) If $F(w)$ be the Fourier transform of a function $f(x)$, then show that the Fourier transform of the derivative of $f(x)$ is $-jw\,F(w)$. 3

(b) If $w=f(z)=\dfrac{1+z}{1-z}$, find (i) $\dfrac{dw}{dz}$ and (ii) determine where $f(z)$ is non analytic. 2

(c) Solve one dimensional heat equation 5

$$\frac{\partial U(x,t)}{\partial t}=h^2\frac{\partial^2 U(x,t)}{\partial x^2}$$

Using Fourier transform. Given the initial condition $u(x,0)=f(x)$.

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